

COMPUTER-AIDED TRANSFORMATIONAL ANALYSIS WITH TONE SIEVES

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ABSTRACT

Sieve-theoretical methods are one of the first historical examples of theoretical tools whose implementational character has largely contributed to the development of computational musicology. According to Xenakis' original intuition, we distinguish between elementary transformations of sieves and compound ones. This makes sense if the sieve construction is considered as part of the musical meaning, as we show by analyzing Scriabin's Study for piano Op. 65 No. 3. This clearly suggests that the transformational character of sieve-theory is still open to new possible applications in computer-aided music analysis.

1. INTRODUCTION

According to Iannis Xenakis introductory note to the cello piece *Nomos Alpha* (1966), sieve theory is "a theory which annexes the residual classes and which is derived from an axiomatics of the universal structure of music". It applies to the formalization of traditional scales as well as microtonal scales, non octaviant scales and any musical phenomenon having a total order structure (intensities, durations, densities, etc.). For example, by combining different periodicity by means of classical set-theoretical operations (union, intersection, complementation, symmetric difference), and by interpreting the resulting sieve in the rhythmic domain one can easily "build [...] very complex rhythmic architectures which can even simulate the pseudo-unpredictable distribution of points on a strait line, if the period is long enough"[16]. In fact, as pointed out by the composer in his *Formalized Music*, "sieve theory is the study of the internal symmetries of a series of points either constructed intuitively, given by observation, or invented completely from moduli of repetition" [17]. Moreover, as the composer already predicted in his thesis defense *Art/Sciences Alloys*, sieve theory is entirely implementable and one of the future research area will be the computer-aided exploration of the theoretical and analytical aspects of this approach [14]. By analyzing the evolution of computational musicology, starting from André Rotté and Marcel Mesnage computer-aided models of music analysis (see [12] for a collected essay of their theoretical writings), many attempts have been made to apply sieve-theory to other dimensions than pitch [2] and to

propose general sieve-theoretical algorithms for the formalization of musical structures (see [16] for some algorithms proposed by Xenakis and [4] for the most recent account of implementational model of sieve-theory). More generally, is the approach of inner metric analysis as such sieve-related. It has been proposed by Guerino Mazzola in the context of the software RUBATO [7] and has been further elaborated and discussed in many musical analyses by Anja Volk (Fleischer)[6] and [13]. The building stones of these analyses are local meters, i.e. bounded elementary sieves of onsets within a piece. The inner metrical analysis is the combinatorial investigation of a complex union of all maximal local meters, i.e. as a compound sieve. Metrical and spectral weights quantify the incidence relation of the bounded or unbounded components, respectively. Section 2 of [8] gives a sieve-theoretic account to the study of musical meter. In this paper we only focus on the pitch domain and on the computer-aided sieve-theoretical description of chord structures and transformations between them.

2. TONE SIEVES AND THEIR TRANSFORMATIONS

The elementary building stones of Xenakis' *sieves* are discrete affine lines of the kind $a_b = \{ka + b, k \in \mathbb{Z}\}$, i.e. arithmetic sequences of integers. General sieves are built from these elementary ones through the boolean operations of union, intersection and complement. *OpenMusic* visual programming language [1] offers specialized functions and factories to construct sieves and to experiment with them for compositorial or analytical purpose (see section 4).

Our analytical example in section 3 departs from two types of elementary sieves and their complements. One the one hand we consider the (complementary) whole-tone sieves

$$\begin{aligned} 2_0 &= \{\dots, -4, -2, 0, 2, 4, \dots\} \\ 2_1 &= \{\dots, -3, -1, 1, 3, 5, \dots\} \end{aligned} \quad (1)$$

On the other hand we consider the elementary minor-third sieves in associations with their octatonic complements

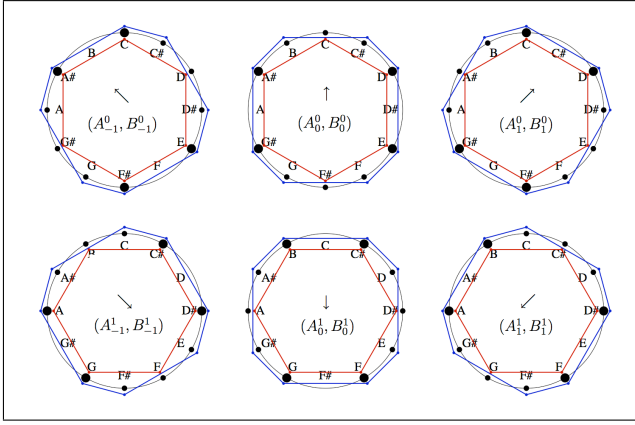


Figure 1. Six configurations of sieve pairs (A_n^m, B_n^m) with elementary wholetone and octatonic sieve components

$$\begin{aligned} 3_{-1} &= \{\dots, -4, -1, 2, 5, 8, \dots\} \\ 3_0 &= \{\dots, -6, -3, 0, 3, 6, \dots\} \\ 3_1 &= \{\dots, -5, -2, 1, 4, 7, \dots\} \end{aligned} \quad (2)$$

$$\begin{aligned} 3_{-1}^c &= \{\dots, -6, -5, -3, -2, 0, 1, 3, 4, \dots\} \\ 3_0^c &= \{\dots, -5, -4, -2, -1, 1, 2, 4, 5, \dots\} \\ 3_1^c &= \{\dots, -4, -3, -1, 0, 2, 3, 5, 6, \dots\} \end{aligned} \quad (3)$$

From these building stones we go on to construct the following unions and intersections:

$$\begin{aligned} A_{-1}^0 &= 2_0 \cap 3_{-1}^c & A_0^0 &= 2_0 \cap 3_0^c & A_1^0 &= 2_0 \cap 3_1^c \\ A_{-1}^1 &= 2_1 \cap 3_{-1}^c & A_0^1 &= 2_1 \cap 3_0^c & A_1^1 &= 2_1 \cap 3_1^c \end{aligned} \quad (4)$$

$$\begin{aligned} B_{-1}^0 &= 2_0 \cup 3_{-1}^c & B_0^0 &= 2_0 \cup 3_0^c & B_1^0 &= 2_0 \cup 3_1^c \\ B_{-1}^1 &= 2_1 \cup 3_{-1}^c & B_0^1 &= 2_1 \cup 3_0^c & B_1^1 &= 2_1 \cup 3_1^c \end{aligned} \quad (5)$$

We consider the six sieve pairs which are obtained from the same two components and introduce the following arrow-notation.

$$\begin{aligned} \nwarrow &= (A_{-1}^0, B_{-1}^0) & \uparrow &= (A_0^0, B_0^0) & \nearrow &= (A_1^0, B_1^0) \\ \swarrow &= (A_{-1}^1, B_{-1}^1) & \downarrow &= (A_0^1, B_0^1) & \searrow &= (A_1^1, B_1^1) \end{aligned}$$

Figure 1 displays these six configurations, which are obtained from the combinatorics of 2 wholetone scales and 3 octatonic scales.

This array is useful for the distinction between elementary and compound sieve transformations. Horizontal and vertical connections correspond to the rotation (transposition) of either the octatonic or the whole tone sieves, respectively. Diagonal connections involve a simultaneous rotation of both components. The following analysis of a late piano study of Alexander Scriabin has the interesting property that all successive sieve transformations are elementary.

bars	1 - 3	4	5 - 8	9 - 11	12	13 - 16
sieves	\downarrow	\swarrow	\searrow	\swarrow	\searrow	\downarrow

Table 1. Sieve-content of bars 1-16



Figure 2. Bars 1- 6 of Scriabin's study for piano Op. 65 No. 3

3. AN ANALYTICAL EXAMPLE

Scriabin's Study for piano Op. 65 No. 3 can be nicely interpreted in terms of the sieve pairs (A_n^m, B_n^m) and the elementary transformations between them. The association between segments of the piece with these sieve pairs is straight forward and from there the transformational analysis leads to a two-voice "Sieve Counterpoint".

3.1. Bars 1 - 16

Bars 1 - 6 exemplify three sieve pairs, namely $\downarrow = (A_0^1, B_0^1)$ in bar 1 - 3, $\swarrow = (A_1^1, B_1^1)$ in bar 4 and $\searrow = (A_{-1}^1, B_{-1}^1)$ in bars 5 - 6 (continuing till bar 8). See Figure2. The left hand of these segments exemplifies the intersection sieves $A_0^1 = \{1, 5, 7, 11\}$, $A_1^1 = \{3, 5, 9, 11\}$, and $A_{-1}^1 = \{1, 3, 7, 9\}$. Both hands together exemplify the sieves B_0^1 , B_1^1 , and B_{-1}^1 up to two missing tones each. The score in Figure 3 displays a reduction of the bars 1 - 16 , which justifies the sieve pairs in table 1.

3.2. Bars 17 - 62

It follows a longer passage of 14 bars, which is associated with the opening sieve pair sieve pair (A_0^1, B_0^1) . The A in bar 20 does not belong to the intersection sieve A_0^1 , but it imitates the A's in bars 17 and 19 and can therefore be seen as a satellite to the right hand. The tones of both

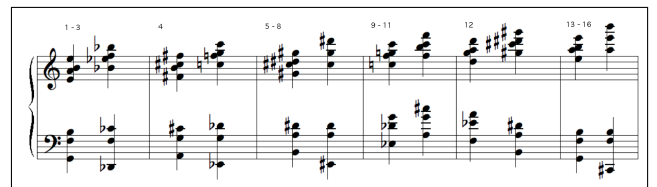


Figure 3. Bars 1- 16 of Scriabin's study for piano Op. 65 No. 3

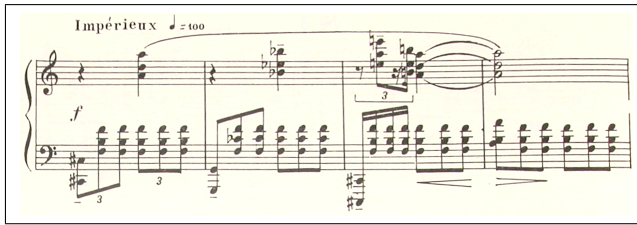


Figure 4. Bars 17 - 20 of Scriabin's study for piano Op. 65 No. 3

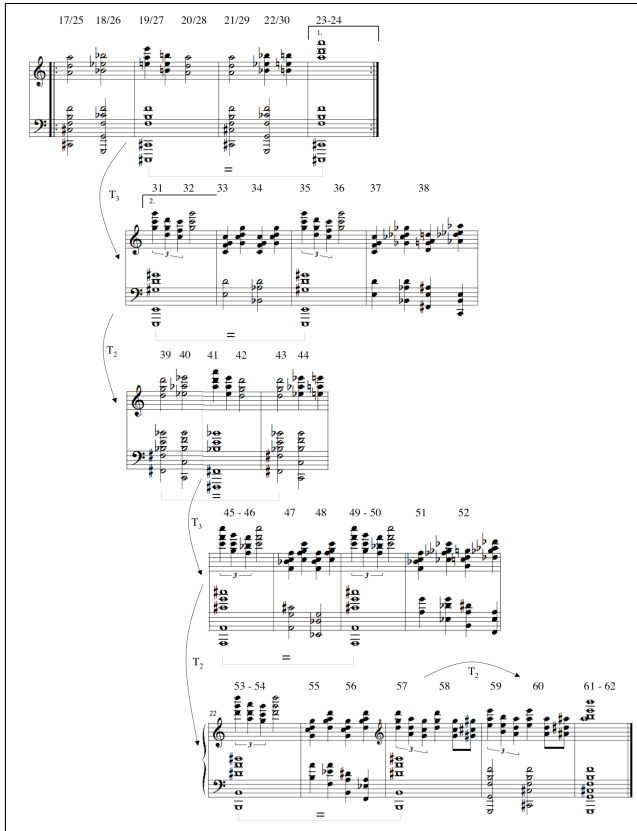


Figure 5. Reduction of bars 17 - 62

hands together still do not form the complete sieve $B_0^1 = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$. But now only one tone is missing: $8 = A_b$. Bars 17 - 20 illustrate the syntactic situation (see Figure 4).

The associated sieves can be verified with the help of the reduction of bars 17 - 62 (see Figure 5).

3.3. Bars 63 - 92 and Coda

Up to a rhythmic detail bars 63 - 92 entirely repeat bars 1 - 30. Thus we have the sieve segmentation in table 2.

The Coda (bars 95 ff.) presents a particularly inter-

bars	63-65	66	67-70	71-73	74	75-78	79-92
sieves	↓	↙	↘	↙	↘	↓	↓

Table 2. Sieve segmentation for bars 63-92



Figure 6. Bars 95 - 97 of Scriabin's study for piano Op. 65 No. 3

bars	95	96	97	98	99 - 102
sieves	↓↑	↓↑	↓↑	↓↑	↓

Table 3. Table captions should be placed below the table

esting situation, because of the chromatic run in the right hand, which seems to undermine the fine harmonic structure by a purely melodic mechanics. However, this is not the case.

It appears that the trioles in each half bar fit with the left hand chords which themselves descend in minor thirds along the four bars 95 - 98. This results in a corresponding pendulum between the sieve pairs $\downarrow = (A_0^1, B_0^1)$ and $\uparrow = (A_0^0, B_0^0)$. Within this process each of the two 10-tone-sieves $B_0^1 = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\}$ and $B_0^0 = \{0, 1, 2, 4, 5, 6, 7, 8, 10, 11\}$ is fully accumulated.

3.4. A Two-Voice Sieve Counterpoint

Aside from the pure segmentation it is of course interesting to study the transformational behavior of the sieves in their succession. To that end we use the metaphor of a two part counterpoint. Each sieve pair is determined by one out of three states of the octatonic component and by one out of two states of the whole tone component (see Figure 7). The following two voice counterpoint encodes the octatonic states in its upper voice (using the tones c_2 , b_1 , and d_b2 for 3_0 , 3_1 , 3_{-1} respectively) and the whole tone states in its lower voice (using the tones c_1 and d_1 for the 2_1 , 2_0 respectively). The sieve-pair "arrow down" = (A_0^1, B_0^1) shall be called the central sieve pair and is meant to be a sieve-theoretic analogue to the traditional concept of "Klangzentrum". In the abstract sieve-counterpoint the concrete tones c_1 and c_2 represent the elementary constituents of the central sieve pair, i.e. the whole tone sieve $\{1, 3, 5, 7, 9, 11\}$ and the octatonic sieve $\{1, 2, 4, 5, 7, 8, 10, 11\}$. In order to avoid confusion between the concrete music and the analytical abstraction we chose tones which are not elements of these sieves. We chose the stable interval of the octave $c_1 - c_2$ in order to express the aspect of centrality, while the other four intervals $c_1 - b_1$, $c_1 - d_b2$, $d_1 - c_2$, $d_1 - b_1$ represent "out of center"-sieve pairs. The sieve-pair (A_1^0, B_1^0) corresponding to the sixth possible interval $d_1 - d_b2$ does not occur in the analysis.

As one can immediately observe, all transformations are elementary, i.e. in each succession there is only one voice moving. This indicates the absence of semitone

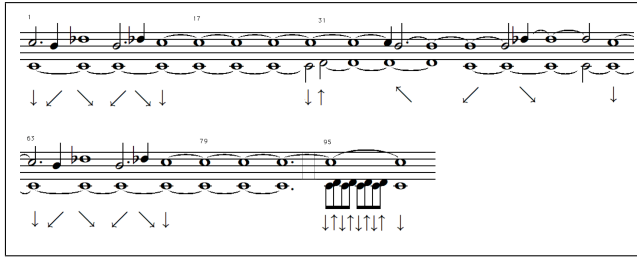


Figure 7. Two-Voice Sieve Counterpoint of the whole piece. The upper voice represents octatonic sieves, the lower voice represents whole tone sieves

and fifth-transpositions between the sieves throughout the piece.

Cliff Callender [5] argues on the background of investigations into voice leading that the harmonic vocabulary of the late compositions of Alexander Scriabin is located between the wholetone scale and the octatonic. This directly motivates the present study. For further investigations see [9], [10] and [11]. In [10] the authors give an informal introduction to the study of transformational logics, which includes sieves of transformations. The paper Noll [9] presents a more mathematically oriented investigation into this subject and, finally, [11] investigates the links between sieves of tones, such as in this paper, with sieves of triadic transformations.

We divide the piece in small harmonic segments such as half bars and sometimes larger segments (as in bar 17 and following bars). To each harmonic segment we may attribute exactly one pair of indices m and n such that the left hand tones are contained in the corresponding intersection sieve A_n^m and that the tones of both hands together are contained in the corresponding union sieve B_n^m . The sieves A_0^1/B_0^1 represent the *Klangzentrum* of this piece. According to the fact that the whole-tone and the octatonic sieves share a periodicity of 12 we may reduce this analysis to pitch classes.

Figure 8 displays the global harmonic organization of the whole piece. For each harmonic segment there is exactly one pair of indices n and m , such that the union sieve A_n^m covers the all pitch classes of both hands and the intersection sieve B_n^m covers the left hand pitch classes. The harmonic organisation of the piece becomes transparent.

This segmentation is a proper refinement to the segmentation into maximal sieve-extensions. The former one has less dense segments within the sieves but it is more sound with the topos-theoretic considerations of [9] as well as with the voice leading considerations of [5].¹

4. SIEVE CONSTRUCTIONS IN *OPENMUSIC*

We now present some aspects of a recent implementation of sieve-theoretical models in *OpenMusic* visual programming language [1]. This environment for computer-

¹ The *OpenMusic* presentation includes an maquette, where each small or large segment can be played and interactively investigated.

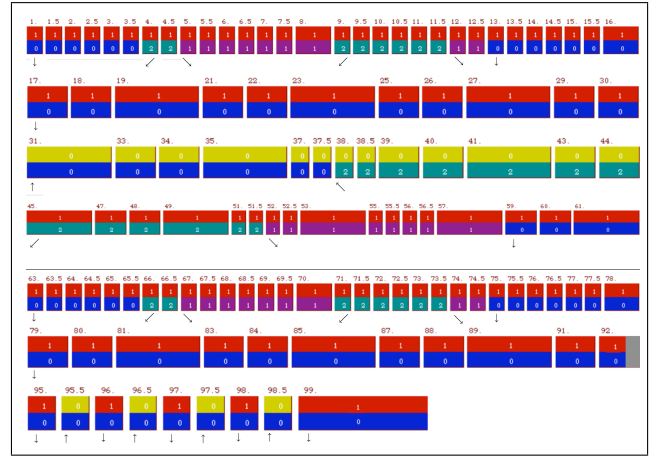


Figure 8. Overview of the analysis of the whole piece

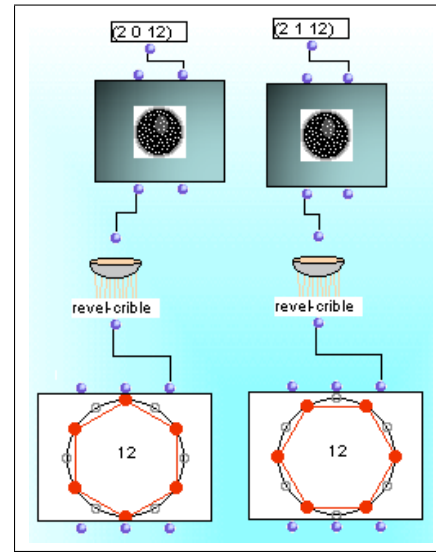


Figure 9. OpenMusic implementation of the complementary whole-tone sieves.

aided music theory, analysis and composition has been integrated as a package of mathematical tools (MathTools) in the last version 5.0 of *OpenMusic*. In a more general way, the MathTools environment enables the construction of algebraic models of music-theoretical, analytical and compositional processes. Its "paradigmatic" architecture, taking several different group actions as the basis of variable catalogues of musical structures, enables to give a formalized and flexible description of the notion of "musical equivalence".

This makes use of some standard algebraic structures (cyclic, dihedral, affine and symmetric groups) as well as more complex constructions based on the ring structure of polynomials. In this package, there are six main families of functions, which are: circle, sieves, groups, sequences, polynomials, canons. In a previous paper [3] we focused on four families of tools which were strictly connected with the problem of paradigmatic classification of musical structures (the circular representation, groups and polynomials).

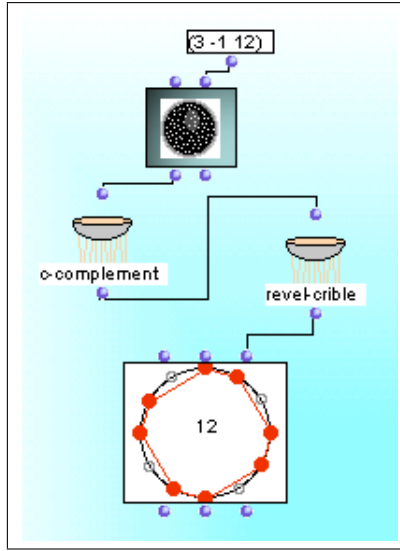


Figure 10. OpenMusic implementation of the octatonic scale.

Although from a mathematical point of view sieves are infinite ordered structures, the sieve theoretical construction we used for the analysis of Scriabin's *Study* Op. 65 No. 3 are isomorphic to subsets of the finite cyclic group of order 12. For this reason, we can easily represent the sieves by means of the circular representation. Figure 9 shows the *OpenMusic* implementation of the complementary whole-tone sieves of equation (1).

Figure 10 shows the constructions and musical representation of the first octatonic sieve in equation (3) starting from its minor-third complements. Notice that the same octatonic sieve could be constructed as the set-theoretical union of two minor-thirds sieves (Figure 11).

By using set-theoretical intersections and unions, we can graphically represented the process leading, for example, to the construction of $A_0^0 = 2_0 \cap 3_0^c$ and $B_0^0 = 2_0 \cup 3_0^c$ (see Figure 12).

Starting from the circular representation, sieves can also be represented in traditional musical notation via the function `c2chord` which maps the geometric representation of a given chord into a chord or a rhythmic pattern. Figure 13 shows the pitch and rhythmic representation of the sieve $B_0^0 = 2_0 \cup 3_0^c$.

5. CONCLUSIONS

It is very likely that the sieve analysis of the chosen example by Alexander Scriabin does not represent a poetic perspective. But on a neutral level of analysis it is quite convincing and pedagogically more convincing than some of Xenakis own examples. Furthermore it suggests a more systematic study of partial transformations in complex sieve constructions, i.e. the independent transformations of elementary components of compound sieves. In our examples the partial transformations represent a special case of transpositions, but generally this will not be the case: A transposition of a defining component of a

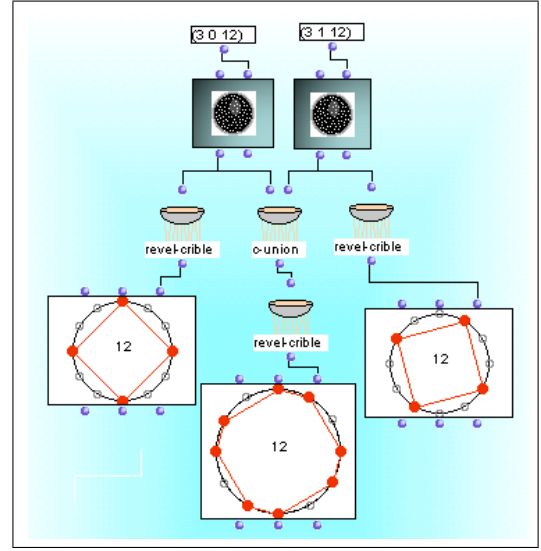


Figure 11. A different set-theoretical construction of the octatonic scale.

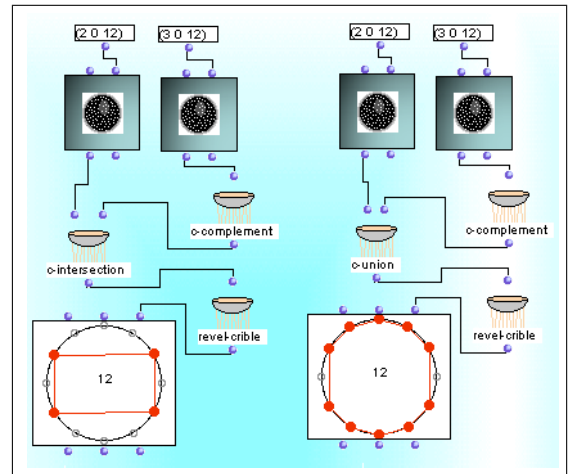


Figure 12. OpenMusic implementation of the sieves $A_0^0 = 2_0 \cap 3_0^c$ and $B_0^0 = 2_0 \cup 3_0^c$.

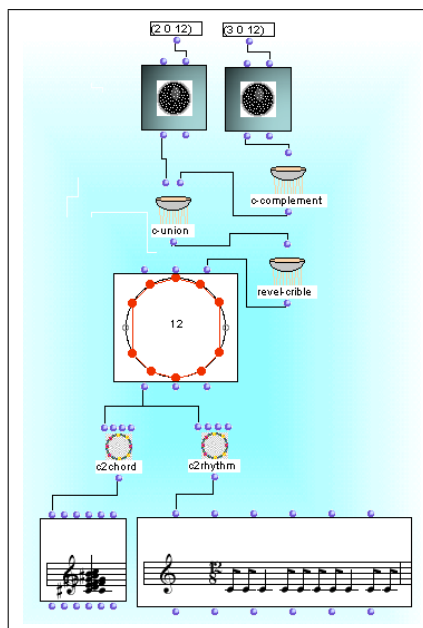


Figure 13. Pitch and rhythmic representation of the sieve $B_0^0 = 2_0 \cup 3_0^c$.

compound sieve does not necessarily result in a transposition of the compound sieve.

Sieve-theoretical models have both a pedagogical and a musicological interest for they enable the music theorist to visualize some structural musical properties in a geometric way and to test the relevance of different segmentations in music analysis. This could have a strong implication in the way to teach music theory, analysis and composition.

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