Analysis and reproduction of the frequency spectrum and directivity of a violin

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Abstract

This research was adopted as an internship at IRCAM, Paris. Many groups in the world, are, like the institute, interested in the combination of electronic music and acoustic instruments. A violin was taken as an example for the acoustic instrument, because of its typical frequency-dependent sound radiation. Previous research has shown that different resonances, or modes, of the violin body cause this radiation pattern.

To determine the radiation pattern and resonance frequencies, an inverse measurement method is adopted as has been described by Weinreich[3]. In this point of view, the violin was excited externally by a loudspeaker and the vibrations at the violin bridge were picked up by a single calibrated piezo-electric sensor. The results were analysed by a simple peak-searching algorithm and by spatial Fourier decomposition, expressing the violin characteristics in a limited number of parameters.

A cube with 6 digitally processed loudspeakers, called *La Timée* applies the 3D reproduction of the radiation. The system is capable of producing a sound field corresponding to all possible combinations of the spherical harmonics of order 0 and 1.

The sound signals are generated by a physical modeling synthesis engine based on the modal theory, called *Modalys.* This software simulates the vibrations of the different modes of the violin parts, like the strings, bridge and also the violinist's fingers. A vibrating plate represents the resonant body; its parameters for the resonance frequencies, damping and radiation patterns have been matched to the measurements. A tune is made in the program and played on the loudspeaker system.

The results were promising, though a disproportionate rate of frequencies above 2kHz was seen in the measured spectrum. Furthermore, it showed that combinations of monopoles and dipoles up to 1.8kHz could describe the characteristics. This is also approximately the maximum frequency that the used version of *La Timée* can handle, so no information was lost. However, the radiation of the violin is also interesting for frequencies above 1.5kHz, so further development is recommended to reproduce higher frequencies.

1. Introduction

The violin has been the topic of a lot of research over the past few hundred years [1]. Its characteristic sound does not have an omnidirectional radiation, but a directivity pattern depending on the frequency. This sound 'lights' the acoustical properties of the surroundings, for example the room or the concert hall, giving an extra dimension to the live performance.

The directivity pattern of a music instrument has been synthesised on for example a Wave Field Synthesis system [2], but not yet on the Spherical Harmonic system of IRCAM. The goal of this research is thus to link the reproduction system to the characteristics of a natural instrument. In more details, the study will consist of (a) the characterization of an acoustical radiation transfer function, (b) the identification of the modes [frequencies, losses, shapes and their radiation pattern associated], (c) the exportation to a sound synthesis engine based on physical modeling, and (d) the reproduction of the radiation by a directivity controlled diffusion device. We have to state here, that no extensive search has been done in the literature, so that we emphasised on the feasibility of the proposed method and also that this first study is a preliminary exploration that should give perspectives for following works.

This article consist of four parts. The first part describes the necessary theory, the second parts describes the directivity measurements on a real violin. The third and fourth part deal with the analysis and reproduction of the measured characteristics, respectively.

2. Theory

Measurements

The measurements were not being applied the direct way, where the bridge or string is excited and the sound radiation is measured, but the inverse way, where an external source excites the violin from a certain direction. This method is based on the reciprocity assumption, which states that if the body vibrates strongly due to a sound wave with given frequency and direction, it will radiate sound waves with the same power and direction as this excitation. Weinreich has exhaustively described this method in 1980 [3]. By doing this, the main uncertainty in the assumption is the fact, that a radiated field from the body has the body as its acoustical centre, but the external field has the source as its acoustical centre. We neglected the influence of this shift in sound centre.

From the literature, it is known that the violin has standing wave solutions of the vibrations of its physical structure, which are called the modes of the violin [3,4]. The analysis consisted of finding the peaks in the measured power spectrum. We stress here, that we assumed the peaks to appear at the modal frequencies of the body. This makes a precise analysis of the violin modal shapes superfluous, but will not violate the overall reproduction of the violin sound.

Reproduction

The reproduction is based on the Huyghens theorem. If we have two sources of which the sound fields are equal at a surrounding surface S, then the fields outside this surface at observer point P are equal. See figure 1. Reproducing with a loudspeaker array, the goal is to find functions that minimise the difference between the two fields [5] which finally constitute the set of filters to apply to the loudspeaker array.

This principle is, in formula,

$$\sum \alpha_i \cdot P_i(x, y, z, t) = T(x, y, z, t) + \varepsilon \text{ , at surface S,} \tag{1}$$

with P the acoustic field of the different loudspeakers on S, α_i the complex, frequency-dependent filter coefficients, and T the original field. The reproduction error is given by ε.

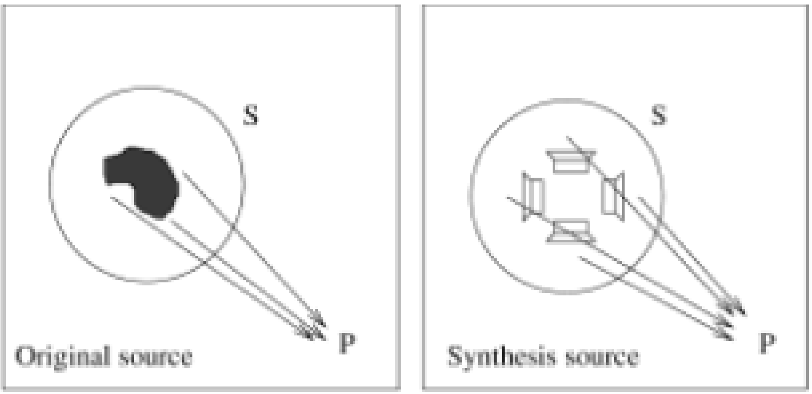

Figure 1. Reproduction of an original field with loudspeakers.

The used reproduction unit has been designed for reproducing the zeroth and first order spherical harmonics. These spherical harmonics are the mathematical solutions of the 3D Laplace equation in spherical co-ordinates.

This method is applied from a pragmatic point of view and results in four sets of filters for the whole system, which is shown in figure 2. These four sets of filters are permanently processed and the combination of the elementary directivities with a set of gains g_i on figure 2 obtains the control. See Misdariis[5] for more details about this method.

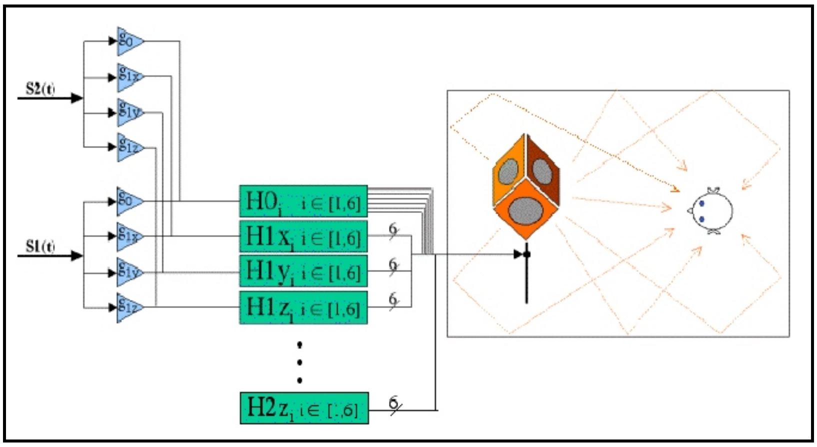

Figure 2. The reproduction signal paths of two different signals S_1 and S_2.

The gains g_i set the monopole and dipole contents of the reproduced field.

Now, the set-up with these filters has one great benefit. If the 3D spatial Fourier decomposition of the sound field is taken, which is nothing more than calculating the resemblance between the field and the spherical harmonics, then the coefficients are directly related to the gains. The formulas that describe this connection are

$$
\begin{aligned}
W &= a_{00}, \\
X &= \frac{1}{\sqrt{2}}\left(a_{11} + a_{1,-1}\right) \\
Y &= \frac{1}{\sqrt{2i}}\left(a_{11} - a_{1,-1}\right) \\
Z &= a_{10}
\end{aligned}
\tag{3}
$$

with W the gain of the monopole, and X,Y, and Z the gains of the 3 dipoles. The *a*'s are the complex spatial Fourier coefficients calculated from the original violin directivity field at a certain frequency.

3. Measurements

One part of this research was the actual measurement of the violin body radiation. The violin was a 70 year old revised study violin.

Set-up

To measure the vibrations of the body, we used a piezo-electric crystal glued between the violin bridge foot and head. We have chosen a commercially available piezo-electric 'pill' of 1.3 mm thickness. This pill was cut into a small slice as wide as the bridge. A bridge was cut into two pieces at exactly the smallest width between the two notches. The head and the foot were then filed to achieve the same height of the bridge as an untouched one when the crystal is glued between the two parts. See figure 3.

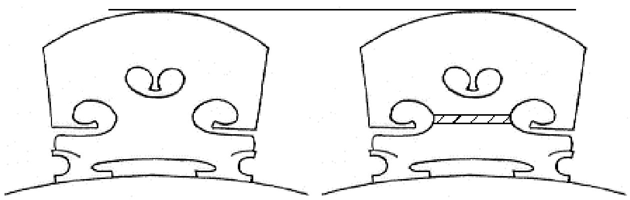

Figure 3. Schematic picture of the placement of the piezo-electric bar.

At this step, we clearly state the fact of using a single sensor, instead of two sensors put at the bottom of each foot of the bridge, like Weinreich [7] did in his experiment. In fact, on this preliminary approach, we are mainly interested in the reproduction procedure of the violin sound so that the actual goal is to pick up a relevant signal of the instrument and play it back with a transfer function that takes into account its spatial behaviour: in this case, we can justify our choice by the consideration that using two different signals would lead to a more complex situation in terms of calibration, mixing, … etc. But, on the other hand, this pragmatic approach of the problem can also be valid within the physical modeling context – the large framework where we finally place the study – if one considers the input of the model in accordance with our experimental output. In practice, we neglect, in a first order approximation, the internal movements of the bridge by integrating it in the global movement of the violin top plate.

A B&K 2635 charge amplifier amplified the electric signal. The assumption is that the amplitude of the electric signal will be linear to the force acting between the body and the strings, and we assume this force to be linear with the amplitude of the body vibrations.

To measure the characteristics of the sensor itself, we sandwiched the sensor between a so-called vibrating pot and a contra mass. The characteristics of the set-up itself were measured with a thin-film piezo-electric which was assumed to have a flat spectrum over the audio frequencies.

The external source was a 2-way coaxial loudspeaker box, a Tannoy System 600. To reduce the influence of the characteristics of the loudspeaker, we took a measurement microphone (B&K 2619 with membrane B&K 4149) that captured its specifications. Afterwards, we corrected the measured signals with this reference by division of the complex frequency spectra.

The violin is mounted on a metal pipe-and-clamps construction, such that the it can both vibrate as freely as possible in the frequency range of interest, and is rigidly mounted. Between the clamps and the violin, small pieces of felt were put to reduce the influence of vibrations of the construction. See figure 4. Suspension will always alter the behaviour of the violin. Marshall, however, has shown that this happens anyhow with different players [6], thus this effect is neglected.

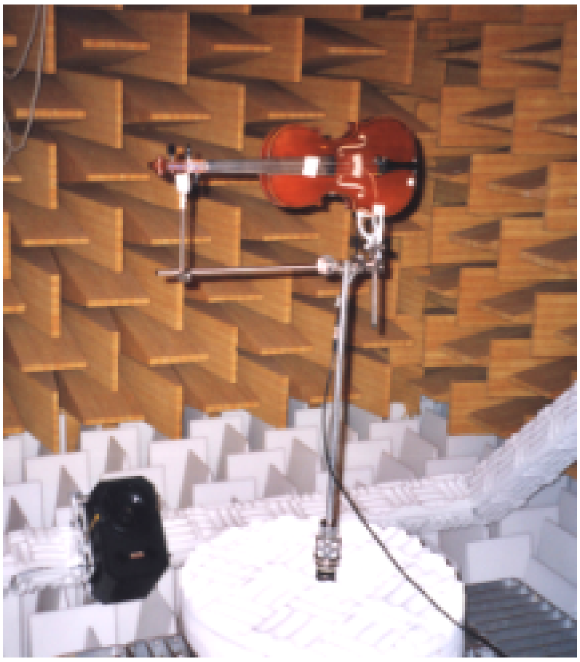

Figure 4. The violin in the anechoic room.

The vertical pipe at the bottom is connected to a B&K turning table.

Furthermore, to reduce the influence of the construction, the reference microphone (that captured the loudspeaker specifications) was mounted on the same construction. This setup was rebuilt twice to check its repeatability, which was quite good.

The construction was attached to a B&K turning table. The loudspeaker was mounted on an elevation arm, which elevated from – 40° to 90°. To cover the whole sphere of 180°×360°, the violin was flipped over its long axis by hand. The distance between the loudspeaker and the sensor was 2.0 m.

The capture software was an IRCAM analysis program, called AMS. This software returned the impulse response of the system in the time domain. The electric cross-talk between the loudspeaker and sensor was eliminated by deleting the first microseconds of the captured signal; in this time interval, the acoustic wave had not reached the violin yet, but only contained the electric cross-talk.

The sampling frequency was 48kHz, with a sample size of 8192 points, giving a frequency resolution of approximately 6Hz. Weinreich has predicted that the directivity field of a violin can change every 44 Hz, due to the resolution of resonance frequencies of the top plate and lower plate of the violin body [4]. Thus, the frequency resolution appeared to be sufficient. The angular resolution was 10°, which was thought to be sufficient.

Spectrum

The spectrum of the violin is depicted in figure 5. As we had an irregularly spaced spatial grid, the spectrum is a weighted sum over all the directional spectra.

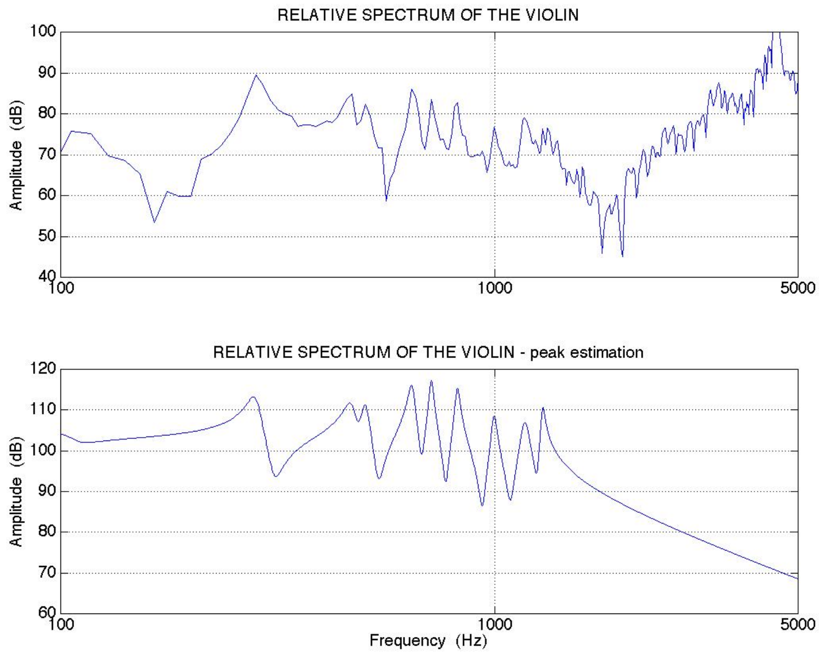

Figure 5. The relative spectrum of the violin.

The bottom spectrum is the relative spectrum of the 10 first estimated peaks (see section Analysis).

The amplitude unit is arbitrary, as the only concern was the relative amplitudes between the subsequent modes. The spectrum was compared with the results from both Weinreich [7] and Marshall [8] for frequencies up to 1 kHz, but as they conclude that each violin has its own frequency response, no exhaustive conclusions can be drawn from the comparison. Moreover, as we used a single sensor experimental set-up we can't find the same results as theirs, because we are not observing exactly the same excitation point of the structure (a single point in the middle of the bridge body vs. two correlated points at the bridge feet), so that we can't detect, for instance, rolling movements of the bridge where each foot acts in contra-phase.

Some directivities are depicted in figure 6. The amplitude of the pressure is constant at the shown surface.

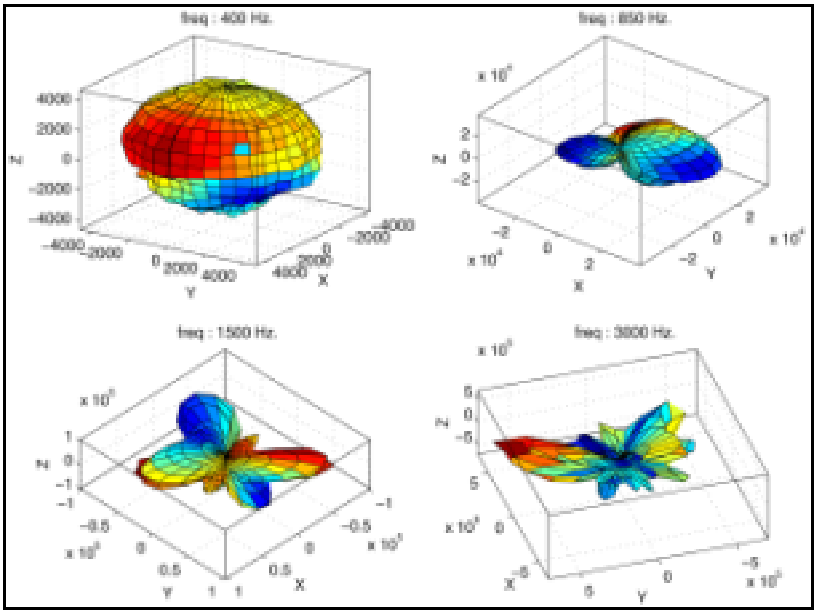

Figure 6. Some shapes of the directivity of the violin.

The f-holes point towards the positive x-axis, the fingerboard lies along the positive y-axis.

From figure 5, it shows that the frequency response is stronger for frequencies above 3kHz. We are not sure about the cause, but suspect that the piezo-electric sensor had a very good acoustic coupling with the higher-frequency body vibrations that was absent in the calibration setup. Thus, an overcompensation of those frequencies might have occurred.

4. Analysis

The analysis is done in two steps. The first step is to find the values of the modal parameters which are the relative amplitude, the frequency and the damping. The second step is to analyse the directivity of the modes. Both these modal characteristics are needed by the reproduction software.

Frequency analysis

A simple peak-searching algorithm found the resonance frequencies, and the height of the peak was the relative amplitude. As we were only interested in frequencies up to 2kHz, we neglected the higher frequencies. This limit was forced by the reproduction system, which could not deal with frequencies higher than 2kHz.

The width of a peak defined the damping. From the theory of damped oscillations, [9], the quality factor Q is given by:

(2)

where f_n is the unforced resonance frequency, and f_1 and f_2 are the two –3dB frequencies. This formula assumes $Q^2 >> 4$, which holds for all peaks. The loss factor was calculated as 2/Q [9].

To check the ability of the suggested method to reproduce the power spectrum, the found values of the resonances are inserted in the forced resonance equation [4], and added. This yields the overall modeled frequency response. For convenience, this response is shown in figure 5 at the bottom, for the ten first peaks detected. The high frequencies are left out in the synthesised spectrum.

Directivity analysis

The directivity of the violin in the total frequency range has been found by the spatial Fourier decomposition.

The power involved in the zeroth and first order is a sum over the squared coefficients a_{ij},

$$P = \sum_{i=0}^{1} \left\| a_{ij} \right\|^2, \quad ||j|| \leq i. \tag{4}$$

The applicability of the proposed method can be checked by comparison of the power contents of the monopole and dipoles to the overall power contents. If the ratio was found to be too low, we would have to conclude that the monopole and dipoles could not describe the radiation field. Then, the reproduction should include the quadripole and higher orders as well. The results are shown in figures 7 and 8.

From figure 7, it is clearly seen that up to 700 Hz the monopole dominates the power spectrum, and from 700Hz up to 1.5kHz the monopole and dipole powers are at similar amplitudes. In figure 8, higher frequencies show the diminishing influence of both these harmonics. From this we conclude that above 1.8kHz the directivity can only be described by harmonic orders higher than zero and one.

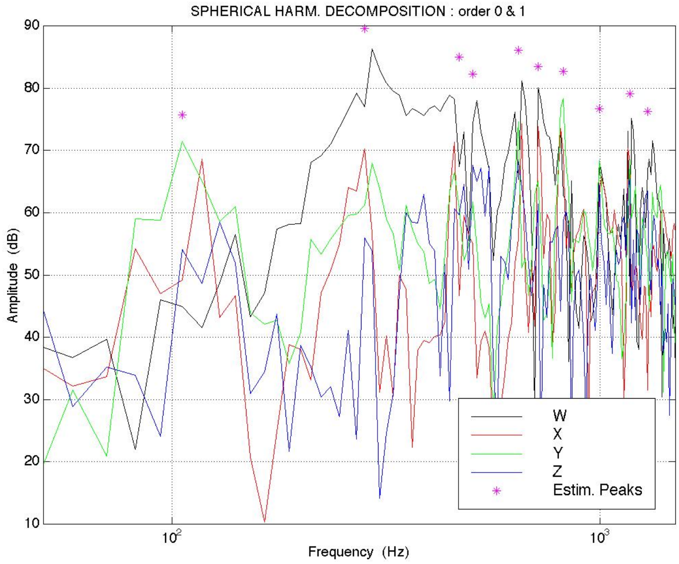

Figure 7. Spectrum of the coefficients of the violin showing its omnidirectional behaviour for frequencies up to 650Hz. The solid line shows the monopole coefficients, the other three lines show the 3 dipole coefficients. The vertical scale is in dB with an arbitrary reference.

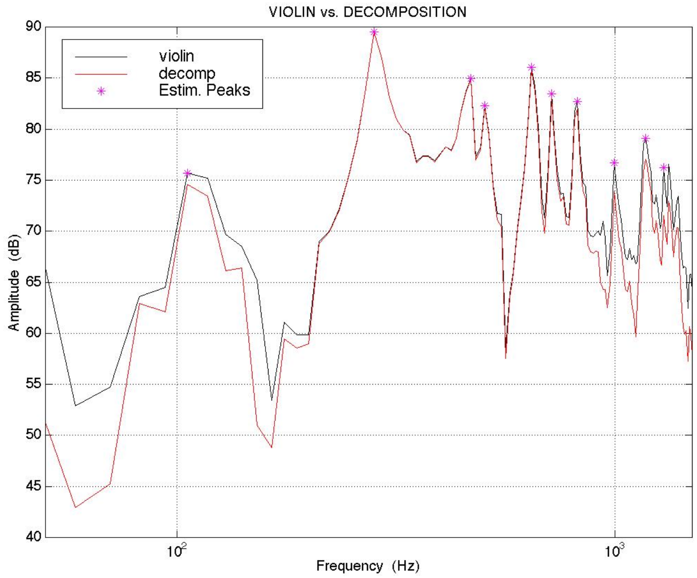

Figure 8. The violin spectrum (black , upper line) and the power of the decomposition in only zeroth and first order harmonics (red, lower line).

5. Reproduction

The reproduction system contains six loudspeakers in a cubic configuration. Due to spatial aliasing, the maximum applicable frequency of the used system is 2kHz. As we have seen, the decomposition of the radiation patterns can be used directly as multiplication factors in the field reproduction. In other words, if the system is fed with four tracks, every track will produce one of the four spherical harmonics. Together they build the 3D sound field.

The sound signal of the violin, which is to be fed into the reproduction system, is made by a modal-based synthesis program called *Modalys*, which decomposes complex structures into simple structures having vibrating modes [10].

Briefly, *Modalys* is based on the decomposition of a complex musical (or any other) instrument into simple structures like plates, strings and tubes. The vibration modes are calculated analytically, for which the parameters and formulas are stored in a database. The structures are connected through physical interactions, like a "pluck", "bow", "hit" or "glue" connection. An actuator is moved, bringing the energy for excitation. The output is the velocity at a user-defined point of the structure. This program is extended with a directivity processing function for the simple structures by the first author. The violin is modeled as having four strings, connected trough a violin bridge glued onto a vibrating plate, which acts as a sound emitting body. Occasionally, a bow strikes the strings, with a "bow" connection (stick-and-loose algorithm). Modeled fingers shorten the strings by pressing the strings to a fingerboard and consequently playing the tune.

Inside this environment the connection to the found parameter values was made. The description of this connection is split into two parts:

- First, the values for the resonances of the vibrating plate are set equal to the values that were found from the measurements: the centre frequency per mode and the damping. We took 10 modes, as this value covered roughly all the peaks in the spectrum. *Modalys* also needs a shape definition of all the single modes, which are not compatible with the 3D modal shapes of a real violin. Therefore, we used the standard square thin plate modes that are available in the *Modalys* database.

- Secondly, the directivity information from the measurements had to be included inside *Modalys*, see figure 9. As we have seen, every mode has its own directivity that is caught into 4 parameters: the contribution of the sound to the monopole W and 3 dipoles X, Y and Z (see equation 3). Thus, the sound signal of every mode of the vibrating plate was multiplied with the four W, X, Y and Z factors, yielding four tracks. These tracks are fed into *La Timée* in the adequate format and played.

A more formal description of the method is the following: if we look at one of the four tracks, then this signal is the weighted sum over the subsequent modes. The four tracks have different weighting coefficients per mode, the coefficients depending on the spherical harmonical contents of that mode.

See figure 9 for the overall scheme; the matrix multiplication is the implementation of the weighted sums.

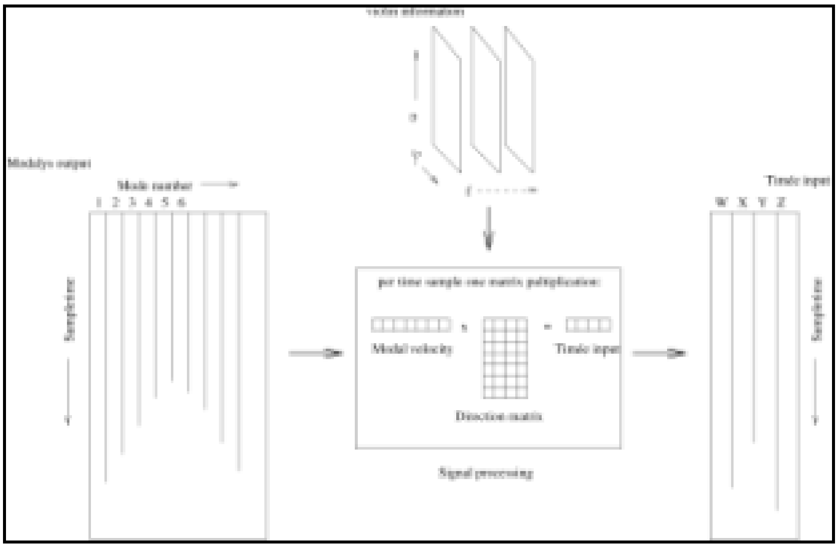

Figure 9. Schematic representation of signal processing.

The played tunes that have been constructed in *Modalys* are a short tango and a plucked-string tune. During the playing of these tunes, the directivity information can just be heard after close listening. The reason for this faint effect was already seen in figure 6. Here, it was shown that only in the range of 0.7 kHz to 1.5 kHz the dipoles do have significant power, but still no more than the monopole.

6. Conclusions

In this research, the spectrum and directivity of the violin body are measured and captured into parameters concerning the frequencies of the body resonances, damping, relative amplitude and directivity information.

Monopoles and dipoles up to 1.8kHz can describe the violin directivity, above this frequency the directivity includes also quadripoles and higher spherical harmonical functions.

This information is used in a reproduction system consisting of instrument modeling software and a cubic loudspeaker set-up. Weighted sums over the modeled modes of the violin body composed four different tracks, which acted as input sound tracks for the reproduced monopole and dipoles.

This research showed that the sound of a violin can be reproduced in all of its aspects, but only for frequencies up to 1.8kHz. The spectrum showed a large amount of high frequencies, and few directivity changes below 0.7kHz. Resonances up to 1.8kHz showed more directivity changes, which could be heard during reproduction stage. The used cubic actuator does not support much higher frequencies and no higher order harmonics, which should be topic of further research.

This research was the first step for the mode-based instrument measurement, characterisation, synthesis and reproduction. A next step can be set to identify all modes of the instrument, using more sensors than one, and exporting the results to the synthesis program in a more complex structure, thus enlarging the resemblance between the violin 3D structure and its physical model.

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